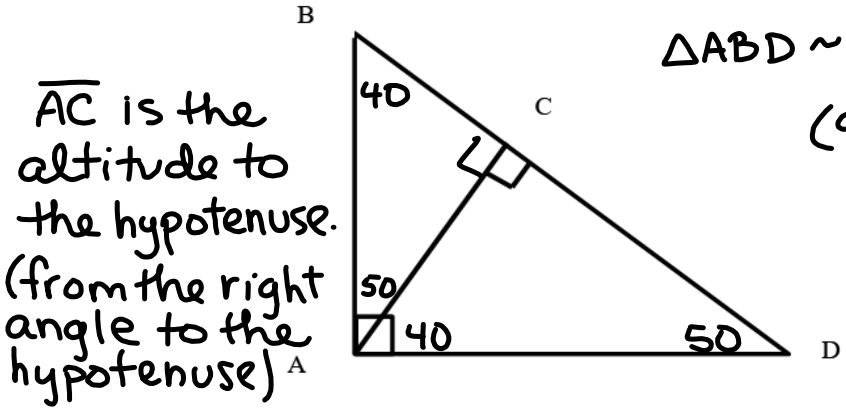
6-2 GEOMETRIC MEAN

SIMILARITY IN RIGHT TRIANGLES

Altitude: The perpendicular height of a geometric figure.

Theorem: The altitude to the hypotenuse of a right triangle forms two triangles that are similar to each other and to the original triangle.



$\overline{AC}$ is the altitude to the hypotenuse. (from the right angle to the hypotenuse)

$\triangle ABD \sim \triangle CAD \sim \triangle CBA$

(90-40-50)

GEOMETRIC MEAN

The geometric mean between two numbers a and b is the positive number **x** where:

$\frac{a}{x} = \frac{x}{b}$ → sometimes referred to as the "mean proportional"

Solve for X:

$x \cdot x = a \cdot b$

$x^2 = ab$

X is considered to be the **geometric mean.**

$\sqrt{x^2} = \sqrt{ab}$

$x = \sqrt{ab}$

Examples: Find the geometric mean of 10 and 40.

$\frac{a}{x} = \frac{x}{b} \rightarrow \frac{10}{x} = \frac{x}{40} \rightarrow x^2 = 400$

$x = \sqrt{400}$

$x = 20$

Find the geometric mean of 1 and 36.

$\frac{1}{x} = \frac{x}{36}$

$x^2 = 36 \rightarrow x = \sqrt{36}$

$x = 6$

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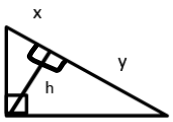
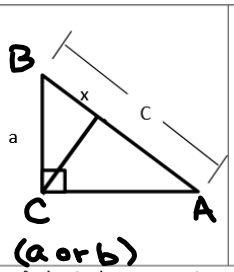
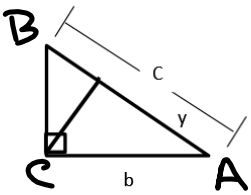
	Altitude and Hypotenuse	Leg and Hypotenuse	
Diagrams			
Corollaries	(h) The length of the altitude is the geometric mean of the lengths of the two segments of the hypotenuse. (x & y)	The length of a leg is the geometric mean of the length of the segment of the hypotenuse that is closest to that leg and the length of the entire hypotenuse.	
Proportions	$\frac{x}{h} = \frac{h}{y}$	$\frac{a}{x} = \frac{c}{a}$	$\frac{b}{y} = \frac{c}{b}$
Equations	$h^2 = xy$ $h = \sqrt{xy}$	$a^2 = cx$ $a = \sqrt{cx}$	$b^2 = yc$ $b = \sqrt{yc}$

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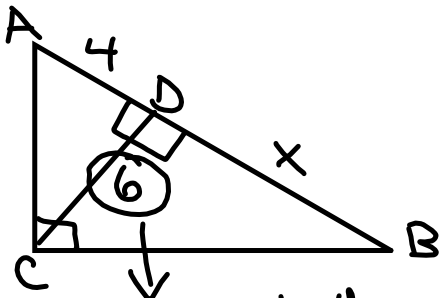
EXAMPLES FOR ALTITUDE AND HYPOTENUSE:

1. In right triangle ABC, altitude CD is drawn to hypotenuse AB. If CD = 6 and AD = 4, find the length of DB. DB = x



Goes into the proportion twice.

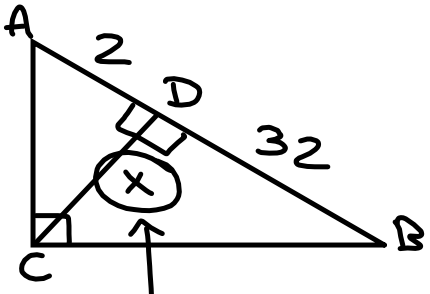
$$\frac{6}{4} = \frac{x}{6}$$

$$\frac{4x}{4} = \frac{36}{4}$$

$$x = 9$$

DB = 9

2. In right triangle ABC, altitude CD is drawn to hypotenuse AB. If AD = 2 and DB = 32, find CD. CD = x



goes into the proportion twice

$$\frac{x}{2} = \frac{32}{x}$$

$$x^2 = 64$$

$$x = \sqrt{64}$$

$$x = 8$$

CD = 8

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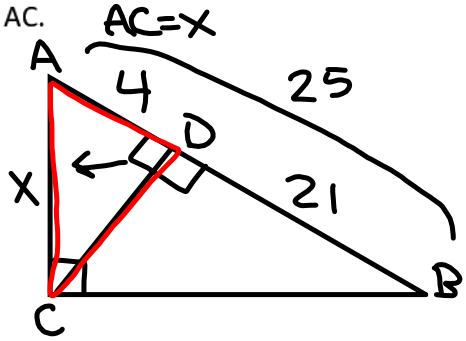
EXAMPLES OF LEG AND HYPOTENUSE: $\frac{BIG\Delta HYP}{BIG\Delta LEG} = \frac{SMALL\Delta HYP}{SMALL\Delta LEG}$

hypotenuse/leg

3. In right triangle ABC, altitude CD is drawn to hypotenuse AB. If AD = 4 and DB = 21, find AC.



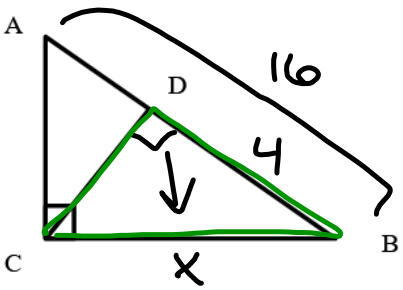
$\frac{25}{X} = \frac{X}{4}$

$X^2 = 100$

$X = 10$

AC=10

4. In the accompanying diagram of right triangle ABC, CD is drawn perpendicular to hypotenuse AB.



If AB = 16 and DB = 4, find BC.

$\frac{16}{X} = \frac{X}{4}$

$X^2 = 64$

$X = 8$

BC=8

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